similar to his, with a provision for carrying the observer similar in general character to his, making it, in fact, exactly on the lines he has laid down.

The first modification would, perhaps, be the best form altogether, as the sides and ends of the outer house could be made with a considerable slope, so as to reduce the upsetting power of the wind. This would retain the important position of the observer as close as possible to the centre of motion of the telescope.

Probable Error of the Clock Correction when both the Clock Rate and the Instrumental Constants are found by a Least Squares Solution of a Single Night's Observations. By the Rev. John T. Hedrick, S.J.

(*Communicated by the Rev. W. Sidgreaves, S.J.*)

It seems to be a common practice in the comparison of time determinations, as in longitude work, to give a constant probable error for the clock correction of each night and to combine the different nights' work according to these probable errors. This is equivalent to taking the clock rate as known perfectly, whereas its value has also its degree of uncertainty, making the probable error of the clock correction quite different for different moments. Of course, if the time comparison takes place during the period of observation, and especially if about its middle, there will be less objection to the method.

A reason for this neglect of the probable error of the clock rate may be the difficulty of determining it when another quite common practice is followed of determining the clock rate from the comparison of the clock corrections found from night to night. If results are expected of sufficient accuracy to make it worth while taking account of probable errors, this practice seems hardly defensible for observations made with portable instruments. For we are likely to have periodic or irregular fluctuations of rate in a clock, not of the best, exposed to receiving injurious shocks in carriage, and not set up so as to be thoroughly protected against the effects of change of temperature, or even against instability in the clock pier. Whatever periodic or accidental errors may exist in the star places of the various modern ephemerides, they are so small that, except in a fixed observatory, the proper method is to find the clock rate each night from that night's observations.

Supposing that a least squares solution of the night's work is thought worth the while, there is still a difficulty in getting *the general expression for the probable error of the clock correction at any moment.* Its probable error at a given moment may be found by making this moment the epoch of the constant part of the total clock correction; but this may bring in practical diffi-

culties in computing, requiring, for example, the use of more decimal places in the coefficients and logarithms.

Let

> ΔT be the clock correction at any moment,
> ΔT_0 the clock correction at some assumed moment, and
> ρ the clock rate.

From the sum of the squares of the residuals after substitution in the equations of condition of the values found by the least squares solution and the coefficients occurring in the solution, we can find the probable error of ΔT_0 and ρ. The probable error of ΔT will be a function of these probable errors and the interval between the epochs of ΔT and ΔT_0, but it will not be a function of these only, if the proper epoch is not chosen for ΔT_0; for in this case the probable error of ΔT_0 will itself depend on that of the rate. (See equations (2) and (3), below.)

Let

$$n', n'', n''' \ldots.$$

be the known terms of the equations of condition after reduction to weight 1. ΔT_0 and ρ are each determined as a linear function of the *ns*.

Let

$$\Delta T_0 = \alpha' n' + \alpha'' n'' + \alpha''' n''' + \ldots.$$
$$\rho = \beta' n' + \beta'' n'' + \beta''' n''' + \ldots.$$

where α', β', &c., are functions of the constant coefficients of the equations of condition.

Now

$$\Delta T = \Delta T_0 + t \, . \, \rho,$$

where t is the interval between the epochs of ΔT and ΔT_0.

Hence

$$\Delta T = (\alpha' + \beta' \, . \, t) n' + (\alpha'' + \beta'' \, . \, t) n'' + (\alpha''' + \beta''' \, . \, t) n''' + \ldots.$$

Let p be the probable error of an observation of weight 1, and $p\Delta T$, $p\Delta T_0$, and $p\rho$ the probable errors of these quantities. We have (Chauvenet, ii., pp. 498, 499, and 516)—

$$(p\Delta T_0)^2 = p^2 \, . \, [\alpha\alpha], \qquad (p\rho)^2 = p^2 \, . \, [\beta\beta],$$
$$(p\Delta T)^2 = p^2 \, . \, [(\alpha + \beta \, . \, t)(\alpha + \beta \, . \, t)] = p^2 \, . \, ([\alpha\alpha] + 2[\alpha\beta] \, . \, t + [\beta\beta] \, . \, t^2).$$

From the degree in t of the parenthesis it is evident that there is a value of t which makes $p\Delta T$ a minimum, and but one value. Let us put $\Delta T_{00} = \Delta T$ for this epoch. It is evident also that if the epoch of ΔT_{00} were chosen as the epoch of ΔT_0, we should have $[\alpha\beta] = 0$; for if it is not zero $p\Delta T$ will not be a

minimum for that epoch. Hence, counting t from the epoch of ΔT_{00}—

$$(p\Delta T)^2 = p^2 \cdot ([\alpha\alpha] + [\beta\beta] \cdot t^2) = (p\Delta T_{00})^2 + (p\rho)^2 \cdot t^2 \quad . \quad . \quad . \quad (1)$$

Hence the probable error of ΔT_{00} is independent of that of the rate. If the epoch assumed for ΔT_0 be not that of ΔT_{00}, and t' be the interval between the two epochs,

$$(p\Delta T_0)^2 = (p\Delta T_{00})^2 + (p\rho)^2 \cdot t'^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$(p\Delta T)^2 = (p\Delta T_0)^2 - (p\rho)^2 \cdot t'^2 + (p\rho)^2 \cdot t^2 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

There is no difficulty in assigning the epoch of ΔT_{00} when only it and ρ enter our equations. It is the mean (simple or weighted) of the times of observation. But we have commonly to determine also the azimuth (a), and in the case of a portable instrument the collimation (c) as well, and perhaps the azimuth after reversal (a'). In this case the epoch of ΔT_{00} is not the mean of the times of observation, and it is the object of this note to show *how to find this epoch from the coefficients formed in solving the normal equations for the epoch of* ΔT_0, or to find the quantity t' of equation (2). The probable error of the clock correction at any moment will then be found by (1) or (3).

Since the squares of the probable errors of the quantities found by a least squares solution are inversely proportional to the coefficients each would have in the final equation if it were the first unknown found, it follows that the moment at which the clock correction has the minimum probable error is the moment which, chosen as the epoch from which to compute the coefficients of the rate in the equations of condition, would bring out ΔT_0 in the final equation with the maximum coefficient.

Let us suppose the most general case, namely, that we have to determine a, a', c, ρ, and ΔT_0, and let w = weight. We have this group of five normal equations, using Gauss's notation.

$$[waa]x + [wab]y + [wac]z + [wad]u + [wae]v + [wan] = 0,$$
$$[wab]x + [wbb]y + [wbc]z + [wbd]u + [wbe]v + [wbn] = 0, \text{ \&c.}$$

Let the order of the unknowns and that of their elimination be that given above.

For $[ee \cdot 4]$, the coefficient of $v(\Delta T_0)$ in the final equation, or the quantity to be made a maximum, we have

$$[ee \cdot 4] = [wee] - \frac{[wae]}{[waa]}[wae] - \frac{[be \cdot 1]}{[bb \cdot 1]}[be \cdot 1] - \frac{[ce \cdot 2]}{[cc \cdot 2]}[ce \cdot 2] - \frac{[de \cdot 3]}{[dd \cdot 3]}[de \cdot 3].$$

In the second member all the terms except the last are independent of d, the coefficient of the rate, and hence of the epoch of ΔT_0, and all are positive. Hence $[ee \cdot 4]$ will be a maximum when the epoch is so chosen as to make the last term zero, or, since $[dd \cdot 3]$ is not infinite, when $[de \cdot 3] = 0$.

We have the following expressions, in determinant notation * :—

$$[de\,.\,3] = |[waa][wbb][wcc][wde]| \div |[waa][wbb][wcc]|,$$
$$[ee\,.\,3] = |[waa][wbb][wcc][wee]| \div |[waa][wbb][wcc]|.$$

Or, since e, the coefficient of ΔT_0 in the equations of condition, $=1$—

$$[de\,.\,3] = |[waa][wbb]]wcc][wd]| \div |[waa][wbb][wcc]|,$$
$$[ee\,.\,3] = |[waa][wbb][wcc][w]| \div |[waa][wbb][wcc]|.$$

The only coefficient affected by a change of epoch for ΔT_0 is d, the coefficient of the rate. The denominator of $[de\,.\,3]$ is independent of d, and, as it is not infinite, to make $[de\,.\,3] = 0$ its numerator must become zero. This numerator expressed in the fuller determinant form is—

$$\text{No. of } [de\,.\,3] = \begin{vmatrix} [waa][wab][wac][wad] \\ [wab][wbb][wbc][wbd] \\ [wac][wbc][wcc][wcd] \\ [wa][wb][wc][wd] \end{vmatrix}$$

Let us suppose that we have formed and solved our normal equations with the epoch T for ΔT_0, and let us change the epoch to $T' = T + t'$. The coefficient of the rate is the time of observation minus the epoch of ΔT_0. We shall have the following relations between the two sets of coefficients, d representing in both sets the coefficient of the rate for the epoch T.

Epoch T,	Epoch $T' = T + t'$
d,	$d - t'$,
$[wad]$,	$[wad] - [wa]\,.\,t'$,
$[wbd]$,	$[wbd] - [wb]\,.\,t'$,
$[wcd]$,	$[wcd] - [wc]\,.\,t'$,
$[wd]$,	$[wd] - [w]\,.\,t'$,

* These expressions, as well as those for the other coefficients occurring in the solution of the normal equations (Chauvenet, ii. pp. 531 and 532), can be found by expressing each as a determinant of the second order; *e.g.*

$$[ee\,.\,3] = \frac{1}{[dd\,.\,2]} \times \begin{vmatrix} [dd\,.\,2][de\,.\,2] \\ [de\,.\,2]\,[ee\,.\,2] \end{vmatrix},$$

substituting in each the determinant expressions for the coefficients of lower order, and applying the theorem

$$\left\| \begin{vmatrix} a_{11}a_{21} \\ a_{12}a_{22} \end{vmatrix} \begin{vmatrix} a_{11}a_{31} \\ a_{13}a_{33} \end{vmatrix} \begin{vmatrix} a_{11}a_{41} \\ a_{14}a_{44} \end{vmatrix} \begin{vmatrix} \cdots \\ \cdots \end{vmatrix} \begin{vmatrix} a_{11} & a_{m1} \\ a_{1m} & a_{mm} \end{vmatrix} \right\| = a_{11}^{m-2}\,.\,|a_{11}a_{22}a_{33}\ldots\ldots a_{mm}|.$$

The first member is the determinant whose element in the kth row and lth column is

$$\begin{vmatrix} a_{11} & a_{(k+1)1} \\ a_{1(l+1)} & a_{(k+1)(l+1)} \end{vmatrix};\ k, l = 1, 2, 3 \ldots\ldots (m-1).$$

Let us substitute the values for the epoch T′ in the numerator of $[de\,.\,3]$. We have

$$\text{No. of } [de\,.\,3] \text{ for epoch T}' = \begin{vmatrix} [waa][wab][wac][wad] - [wa]\,.\,t' \\ [wab][wbb][wbc][wbd] - [wb]\,.\,t' \\ [wac]\,[wbc]\,[wcc]\,[wcd] - [wc]\,.\,t' \\ [wa]\;\;[wb]\;\;[wc]\;\;[wd]\; - [w]\;\;.\,t' \end{vmatrix}.$$

Splitting up this determinant into two and factoring out t' in the second, we have, for the epoch T′—

$$\text{No. of } [de\,.\,3] = |[waa][wbb][wcc][wd]| - |[waa][wbb][wcc][w]|\,.\,t'.$$

Putting this equal to zero and noticing that the first term and the factor of t' in the second term are proportional to $[de\,.\,3]$ for the epoch T and $[ee\,.\,3]$ respectively (4), we find for t' *the interval from the epoch assumed for* ΔT_0 *to that which would give the maximum coefficient for* ΔT_0 *in the final equation, or to the epoch at which the clock correction has the minimum probable error,*

$$t' = \frac{[de\,.\,3] \text{ for the assumed epoch}}{[ee\,.\,3]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The value found for t' is the quotient of the same coefficients, after the equations have been freed from all the unknowns except ΔT_0 and ρ, of which it would have been the quotient, had there been only these two unknowns in the original equations, namely, of the coefficient of ΔT_0 in the normal equation in ρ by that in its own normal equation. It will, in general, differ from $[wde] \div [wee]$ = the mean of the times of observation.

In the example given below the square brackets are used also to indicate the number whose logarithm they enclose.

Summary.

The epoch of the clock correction of maximum weight, or minimum probable error, is not, in general, the mean of the times of observation when, besides the constant clock correction and the clock rate, instrumental constants are also determined from the observations, (5).

If these quantities are found by a least squares solution, this epoch is before or after the epoch assumed for the constant clock correction by an interval which is the quotient of the coefficient of the constant clock correction in the normal equation for the rate by its coefficient in its own normal equation, after the elimination of the other unknowns, (5).

If we count from this epoch, the probable error of the clock correction at any other time is what it would be if the constant correction and the rate were independently observed quantities—

that is, its square is the sum of the square of the probable error at this epoch and the product of the square of the probable error of the rate into the square of the interval from this epoch, (1). Hence the square of the probable error of the clock correction at this epoch is equal to the square of the probable error of the clock correction at the assumed epoch minus the product of the square of the probable error of the rate into the square of the interval between the two epochs.

Example.

Number of stars observed, 20. Number of quantities determined, 5.

Putting v here for the residuals, $[wvv] = 0^s \cdot 0251$.

For an observation of weight 1, $p^2 = 0^s \cdot 000761 = [6 \cdot 88 15]$.

Mean of the times of observation $= 8^h \cdot 98$. Assumed epoch of $\Delta T_0 = 9^h \cdot 0$.

$$[ee.4] = 3 \cdot 28, \quad (p\Delta T_0)^2 = [6 \cdot 8815] \div [0 \cdot 5159] = [6 \cdot 3656] = 0^s \cdot 000232.$$

$$[dd.4] = 4 \cdot 44, \quad (pp)^2 = [6 \cdot 8815] \div [0 \cdot 6474] = [6 \cdot 2341] = 0^s \cdot 000171.$$

$$[de.3] = +5 \cdot 75 = [0 \cdot 7597], \quad [ee.3] = +6 \cdot 85 = [0 \cdot 8357], \quad t' = [9 \cdot 9240] = 0^h \cdot 84.$$

Epoch of maximum coefficient $= 9^h \cdot 84$.

$$(p\Delta T_{00})^2 = 0^s \cdot 000232 - 0^s \cdot 000171 \, . \, (0 \cdot 84)^2 = 0^s \cdot 000111, \quad p\Delta T_{00} = \pm 0^s \cdot 011.$$

$$\text{At time T, } p\Delta T = \pm \sqrt{0^s \cdot 000111 + 0^s \cdot 000171 \, . \, (T - 9^h \cdot 84)^2}.$$

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On the Photographic Magnitude of Nova Aurigæ, as Determined at the Royal Observatory, Greenwich. II. By W. H. M. Christie, M.A., F.R.S., Astronomer Royal.

In the *Monthly Notices* for last March (vol. lii. p. 357) results for the photographic magnitude of *Nova Aurigæ* were given as determined up to March 9. Since then photographs have been taken up to April 1, when the photographic magnitude had fallen below 14, and again at the end of August and beginning of September, when the star had brightened. Further measures were also made of the earlier photographs, modifying slightly the results given in the previous paper. The measures were made and reduced in the manner explained in that paper, the magnitude of the *Nova* being inferred by comparison with four Argelander stars of 8–9 magnitude by means of the formula

$$m = 2 \cdot 5 \, (\log t - 0 \cdot 97 \sqrt{d}) + \text{const.}$$

given in the *Monthly Notices*, vol. lii. p. 146, the measures of diameter of the four comparison stars being used to determine